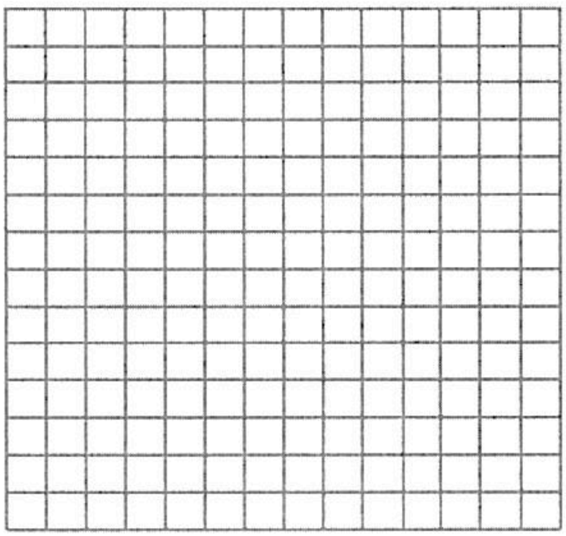
Name:________________________________ Date:__________

A2CC: Logarithmic Functions

One of the properties of the exponential function $f(x) = b^x$ is that it is a 1-1 function. Remember that this means it has an inverse function whose graph can be obtained by reflecting the graph of $y = b^x$ through the line $y = x$.

Let's graph the inverse of the function that we were looking at yesterday, $y = 2^x$.



Write an equation for the inverse of $y = 2^x$.

The equation $x = b^y$ tells us that y is the exponent on b that produces x. In situations like this the word logarithm is used in place of exponent. A **logarithm** is an exponent. We can abbreviate to:

$$y = \log_b x$$

read as "y equals log x to the base b" or "y equals log x base b."

Exponential form: $x = b^y$

Logarithmic form: $y = \log_b x$

General Graphs of Logarithmic Functions

$y = \log_b x, b > 1$

$y = \log_b x, 0 < b < 1$

Properties of Logarithmic Functions

1. The domain consists of all positive numbers.
2. The range consists of all real numbers.
3. The function is increasing (the curve is rising) when $b > 1$, and its is decreasing (the curve is falling) when $0 < b < 1$.
4. It is a one-to-one function.
5. The point (1,0) is on the curve.
6. There is no y-intercept.
7. The y-axis is a vertictal asymptote to the curve.

Practice

1. Evaluate the following logarithms. If needed, write an equivalent exponential equation. (Without the use of your calculator.)

(a) $\log_2 8$ (b) $\log_4 16$ (c) $\log_5 625$ (d) $\log_{10} 100,000$

(e) $\log_6 \left(\frac{1}{36}\right)$ (f) $\log_2 \left(\frac{1}{16}\right)$ (g) $\log_5 \sqrt{5}$ (h) $\log_3 \sqrt[5]{9}$

It is critically important to understand that logarithms **give exponents as their outputs**. We will be working for multiple lessons on logarithms and a basic understanding of their inputs and outputs is critical.

2. If the function $y=\log_2(x+8)+9$ was graphed in the coordinate plane, which of the following would represent its *y*-intercept?

(1) 12 (3) 8

(2) 13 (4) 9

3. Between which two consecutive integers must $\log_3 40$ lie?

(1) 1 and 2 (3) 3 and 4

(2) 2 and 3 (4) 4 and 5

<u>Calculator Use and Logarithms</u> – Most calculators only have two logarithms that they can evaluate directly. One of them, $\log_{10} x$, is so common that it is actually called the **common log** and typically is written without the base 10.

$$\log x = \log_{10} x \quad \text{(The Common Log)}$$

4. Evaluate each of the following using your calculator.

(a) $\log 100$ (b) $\log\left(\frac{1}{1000}\right)$ (c) $\log\sqrt{10}$

5. Can the value of $\log_2(-4)$ be found? What about the value of $\log_2 0$? Why or why not? What does this tell you about the domain of $\log_b x$?

Common Core Algebra II Homework

Fluency

1. Which of the following is equivalent to $y = \log_7 x$?

   (1) $y = x^7$ (3) $x = 7^y$

   (2) $x = y^7$ (4) $y = x^{1/7}$

2. If the graph of $y = 6^x$ is reflected across the line $y = x$ then the resulting curve has an equation of

   (1) $y = -6^x$ (3) $x = \log_6 y$

   (2) $y = \log_6 x$ (4) $x = y^6$

3. The value of $\log_5 167$ is closest to which of the following? Hint – guess and check the answers.

   (1) 2.67 (3) 4.58

   (2) 1.98 (4) 3.18

4. Which of the following represents the y-intercept of the function $y = \log(x + 1000) - 8$?

   (1) −8 (3) 3

   (2) −5 (4) 5

5. Determine the value for each of the following logarithms. (Easy)

   (a) $\log_2 32$ (b) $\log_7 49$ (c) $\log_3 6561$ (d) $\log_4 1024$

6. Determine the value for each of the following logarithms. (Medium)

   (a) $\log_2\left(\frac{1}{64}\right)$ (b) $\log_3(1)$ (c) $\log_5\left(\frac{1}{25}\right)$ (d) $\log_7\left(\frac{1}{343}\right)$

7. Determine the value for each of the following logarithms. Each of these will have non-integer, fractional answers. (Difficult)

(a) $\log_4 2$ (b) $\log_4 8$ (c) $\log_5 \sqrt[3]{5}$ (d) $\log_2 \sqrt[5]{4}$